Pseudoholonomic Behavior of Planar Space Robots

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I. Introduction

SOME of the early research in the area of motion planning and control of space robots can be found in Refs. 1–6 and references therein. More recent work in this area can be found in Refs. 7–9. An interesting problem of space manipulators is the repeatability problem. Because of the nonholonomic constraints of motion, closed joint space trajectories result in nonrepeatable motion of the end effector. Also, closed end-effector trajectories result in nonrepeatable joint motion under pseudoinverse control of the generalized Jacobian.[5] This can be a problem if the robot is required to perform repeated tasks without performing intermediate reorientation maneuvers. A related problem is the repeatability problem in terrestrial redundant manipulators; a complete list of references on this topic can be found in Ref. 10.

The property of repeatability is ensured if the differential constraints of motion of a system are integrable. The integrability of the constraints is, however, a sufficient condition for repeatability, it is not a necessary condition. In this Note we show that there exist some closed paths in the joint space that are holonomic loops along which the space robot exhibits holonomic behavior. When the joints of the space robot move along these trajectories, the end effector traces a closed curve in the workspace. The inverse problem is of greater practical importance where it is desired to find closed trajectories in the workspace that generate closed joint space trajectories under pseudoinverse control of the generalized Jacobian. We address this problem here specific to a two-link manipulator. Our study is primarily based on planar space manipulators; the extension of our approach to space robots in higher dimensions lies within the scope of our future research work.

II. Repeatability Problem

Consider a planar space robot with two links mounted on a space vehicle,[11] as shown in Fig. 1. The coordinates of the end effector x_E and y_E have the functional dependence

$$x_E = f_1(x_0, y_0, \theta_0, \theta_1, \theta_2), \qquad y_E = f_2(x_0, y_0, \theta_0, \theta_1, \theta_2) \quad (1)$$

where x_0 and y_0 are the coordinates of the center of mass of the space vehicle, θ_0 is the orientation of the vehicle, and θ_1 and θ_2 are the joint variables. The motion of the center of mass of the space vehicle, for zero initial linear momentum, can be expressed as

$$x_0 = f_3(\theta_0, \theta_1, \theta_2), \qquad y_0 = f_4(\theta_0, \theta_1, \theta_2) \quad (2)$$

If the initial linear momentum of the system were nonzero, then the variables x_0 and y_0 would explicitly depend on time.

If the orientation of the space vehicle traces a closed curve as the joints move along a closed trajectory, it is clear from Eqs. (1) and (2) that all of the configuration variables including x_0, y_0, x_E, and y_E will move along closed trajectories. This is not true in the general case; when the joints move along closed trajectories and the system maintains zero angular momentum, the change in the orientation of the space vehicle is expressed as

$$d\theta_0 = g_1(\theta_1, \theta_2)\, d\theta_1 + g_2(\theta_1, \theta_2)\, d\theta_2 \quad (3)$$

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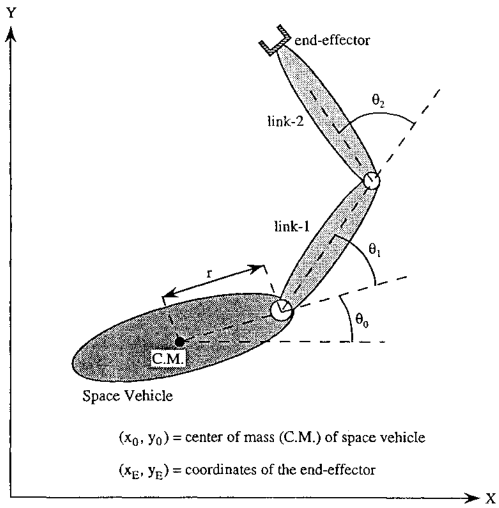

Fig. 1 Planar space robot capable of exhibiting pseudoholonomic behavior.

where g_1 and g_2 (Ref. 11) have not been defined here for the sake of brevity. Using Stokes' theorem,[12] the line integration of $d\theta_0$ along a path ∂D in θ_1 and θ_2 is expressed as

$$\int_{\partial D} d\theta_0 = \int_D \left[\frac{\partial g_2}{\partial \theta_1} - \frac{\partial g_1}{\partial \theta_2}\right] d\theta_1\, d\theta_2 \quad (4)$$

If the constraint given by Eq. (3) were a holonomic constraint, then we would have

$$\frac{\partial g_2}{\partial \theta_1} = \frac{\partial g_1}{\partial \theta_2} \quad (5)$$

Then the change in θ_0 would be zero for all closed paths in the domain D because of the integrable nature of the constraints. This would ensure repeatability. For a planar space robot where Eq. (5) does not hold good, repeatability can still be achieved for specific closed paths in the domain D. Let us define

$$\left(\frac{\partial g_2}{\partial \theta_1} - \frac{\partial g_1}{\partial \theta_2}\right) \triangleq F(\theta_1, \theta_2) \quad (6)$$

The change in the orientation of the space vehicle is equivalent to

$$\int_{\partial D} d\theta_0 = \int_D F(\theta_1, \theta_2)\, d\theta_1\, d\theta_2 = F(\theta_1^*, \theta_2^*) \int_D d\theta_1\, d\theta_2$$
$$= F(\theta_1^*, \theta_2^*)\pi(D), \qquad (\theta_1^*, \theta_2^*) \in D \quad (7)$$

where Eq. (7) was obtained by the application of the mean value theorem of integral calculus. The symbols θ_1^* and θ_2^* denote some point within the domain D, and $\pi(D)$ is the area enclosed within the closed curve ∂D. $F(\theta_1^*, \theta_2^*)$ can be interpreted as the mean value of the function F, defined in Eq. (6), taken over the domain D. If this mean value is zero, we would have a zero net change in the orientation of the space vehicle. This would ensure repeatability in the motion of the end effector for cyclical joint motion. We are now ready to state the necessary condition for the repeatable motion of the space manipulator.

Proposition. A necessary condition for the repeatable motion of the space manipulator system for cyclical joint motion is that the closed path ∂D in the joint space should enclose at least one point where the function F defined by Eq. (6) is equal to zero.

The proof of the proposition is quite straightforward and is left to the reader.

III. Search for Holonomic Loops in the Joint Space

All closed joint trajectories that will ensure repeatability will have to satisfy the necessary condition for repeatability, developed in Sec. II. Therefore, we first look at all points in the θ_1–θ_2 space where the function $F(\theta_1, \theta_2)$ in Eq. (6) is zero. This is shown in Fig. 2 for the set of kinematic and dynamic parameters provided in Ref. 3.

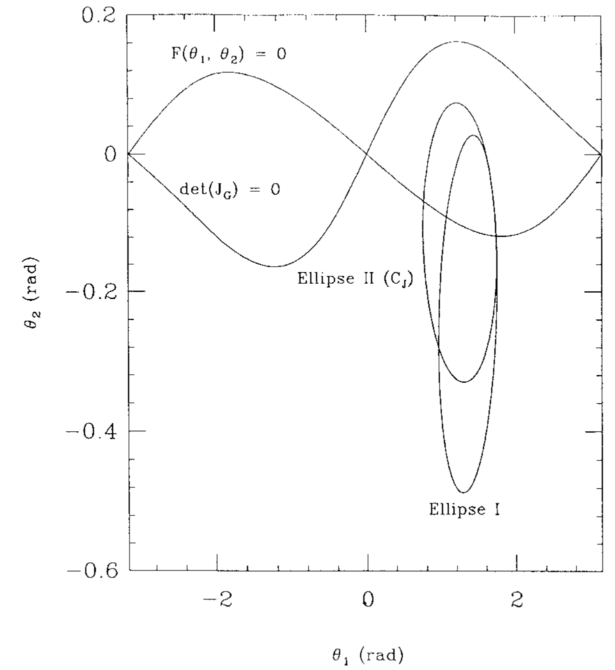

Fig. 2 Elliptical paths in the joint space of the space manipulator; path I is the initially chosen path, and path II is the optimized path.

In a practical situation, we will be required to find a closed path in the joint space passing through some prespecified joint configuration. If this configuration is described by the coordinates θ_{10} and θ_{20}, then we parameterize our elliptical path as follows:

$$\begin{aligned}\theta_1 &= \theta_{10} + a\cos\phi(\cos 2\pi t - 1) - b\sin\phi\sin 2\pi t \\ \theta_2 &= \theta_{20} + a\sin\phi(\cos 2\pi t - 1) + b\cos\phi\sin 2\pi t\end{aligned} \qquad t \in [0, 1] \quad (8)$$

where a and b are the semimajor and semiminor axes of the ellipse and ϕ is the angle between the major axis and the coordinate axis θ_1. The initial choices of these parameters are arbitrary; we only make sure that the ellipse encompasses at least one point where the function F defined by Eq. (6) is zero. Our goal is to change the three parameters of the ellipse so that the value of the surface integral in Eq. (4) reduces to zero. Of the three different parameters a and b are not allowed to change independent of one another because we want to eliminate the trivial solution where the surface integral is zero since the area of the closed path is equal to zero. One simple way to avoid this situation is to impose the restriction that the area of the ellipse is a constant. This is equivalent to the constraint $a\,\mathrm{d}b + b\,\mathrm{d}a = 0$. We define a function V as follows:

$$V = \zeta^2, \qquad \zeta \triangleq \int_D F(\theta_1, \theta_2)\,\mathrm{d}\theta_1\,\mathrm{d}\theta_2 \qquad (9)$$

and solve the unconstrained minimization problem by implicitly assuming that a and b are dependent. Although there are many methods for unconstrained minimization, we choose the simplest method of steepest descent. The steepest direction of descent is computed as

$$\mathrm{d}\phi = -\zeta\frac{\partial\zeta}{\partial\phi}, \qquad \mathrm{d}a = -\zeta\frac{\partial\zeta}{\partial a} \qquad (10)$$

In Eq. (10), the quantities $(\partial\zeta/\partial\phi)$, and $(\partial\zeta/\partial a)$ are computed by numerical partial differentiation. While computing the term $(\partial\zeta/\partial a)$ it has to be remembered that a change in a is accompanied by a change in b given by the constraint already discussed.

The optimization technique just discussed provides us with a systematic way to reach the local minimum value of the function V. If this minimum value is zero, we will have converged upon the holonomic loop or the closed path in the joint space $\mathcal{C_J}$ that will result in closed paths in the workspace $\mathcal{C_W}$.

We conclude this section by reconsidering Eqs. (1–3). These three equations can be combined together to obtain

$$\dot{x} = J_G\dot{\theta}, \qquad x \triangleq (x_E \quad y_E)^T, \qquad \theta \triangleq (\theta_1 \quad \theta_2)^T \qquad (11)$$

In Eq. (11) J_G is the generalized Jacobian.[5] This matrix defines the mapping from the joint space to the workspace. Although the Jacobian relationship in Eq. (11) cannot be integrated to obtain a kinematic function of the form $x = f(\theta)$, the optimization procedure outlined enables us to converge upon an implicit kinematic function that maps the holonomic loop $\mathcal{C_J}$ to the corresponding closed path in the workspace $\mathcal{C_W}$.

IV. Design of Repeatable Trajectories in the Work Space

For an n link planar space manipulator, the dimensions of θ and J_G in Eq. (11) are n and $2 \times n$, respectively. Under resolved motion rate control, the pseudoinverse solution invokes the control law

$$\dot{\theta} = J_G^{\#}\dot{x} \qquad (12)$$

where $J_G^{\#}$ is the pseudoinverse of J_G. Under the assumption that the generalized Jacobian is not at any singular configuration, the null space of J_G will have a dimension of $(n - 2)$. The pesudoinverse solution has the minimum norm property that implies that the joint motion $\dot{\theta}$ obtained from Eq. (12) will have to be orthogonal to the null space of J_G. Since the null space of J_G has a dimension of $(n-2)$, the orthogonality condition imposes $(n - 2)$ velocity constraints.

For the case where $n = 2$ and the generalized Jacobian is full rank, the pseudoinverse solution will impose no constraints; the motion of the system will be governed only by the two kinematic relations given by Eq. (11). From our discussion in Sec. III we know that the Jacobian relationship in Eq. (11), although nonholonomic, is equivalent to the existence of an implicit kinematic function mapping from the holonomic loop $\mathcal{C_J}$ to the corresponding closed path in the workspace $\mathcal{C_W}$. Consequently, the closed path in the workspace $\mathcal{C_W}$ will map back to the closed path in the joint space $\mathcal{C_J}$ under pseudoinverse control.

In summary, a holonomic loop in the joint space $\mathcal{C_J}$ results in a closed path in the workspace $\mathcal{C_W}$. The closed path in the workspace $\mathcal{C_W}$ will map back to the closed path in the joint space $\mathcal{C_J}$ under pseudoinverse control if the generalized Jacobian in Eq. (11) has a null space of dimension zero. This can be achieved when the planar space manipulator has two links and the holonomic loop $\mathcal{C_J}$ does not pass through any dynamic singularities.[3] In Fig. 2 we plot the singular points of the Jacobian of the two link space manipulator.

V. Simulations

The three equations due to the conservation of linear and angular momentum were integrated simultaneously to study the evolution of the dependent variables x_0, y_0, and θ_0. The kinematic and dynamic parameters used in the simulations were taken from Ref. 11. The configuration variables of the robot at the initial point of time were chosen as

$$\begin{gathered} x_0 = 0.00000, \qquad y_0 = 0.00000, \qquad \theta_0 = 0.00000 \\ \theta_1 = 1.75000, \qquad \theta_2 = -0.15000 \end{gathered} \qquad (13)$$

and the parameters of the elliptical path in Eq. (8) were chosen as

$$a = 0.40000, \qquad b = 0.25000, \qquad \phi = 0.20000 \qquad (14)$$

where the units are in radians. For these set of values, the numerical value of the surface integral ζ, defined by Eq. (9), was found to be $\zeta = 1.621 \times 10^{-3}$. The convergence criterion was set at $|\zeta| \leq 1.0 \times 10^{-10}$. The values of the path parameters after convergence were

$$a = 0.49696, \qquad b = 0.20122, \qquad \phi = -0.04531 \qquad (15)$$

The two elliptical paths are shown in Fig. 2. Ellipse I corresponds to the initial choice of the path parameters and ellipse II corresponds to the optimized values of the path parameters. It is clear that the optimized path satisfies the necessary condition for repeatability, but it does not pass through any dynamic singularities. Figure 3 depicts the motion of the end effector of the space robot for 20 cycles of joint motion along the elliptical paths I and II in Fig. 2. The end effector is seen to drift for path I but shows repeatable motion for path II.

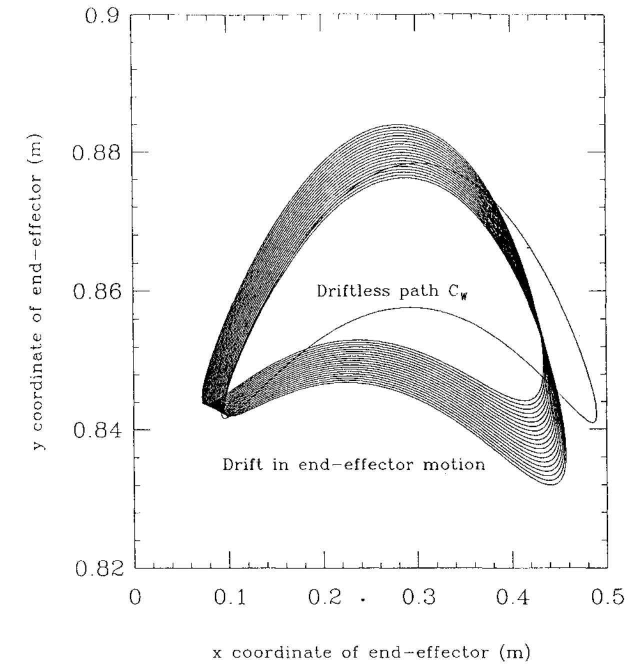

Fig. 3 Path I in the joint space, shown in Fig. 2, fails to produce a closed path in the workspace of the space manipulator; path II produces repeatable motion.

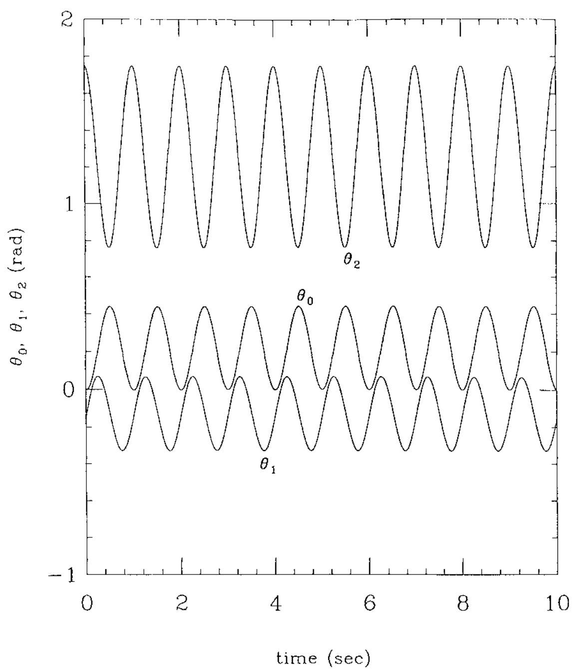

Fig. 4 Evolution of the orientation of the space vehicle and the joint angles of the space manipulator as the manipulator tracks the closed end-effector trajectory in Fig. 3 for 10 cycles using pseudoinverse control of the generalized Jacobian.

This closed path is $\mathcal{C}_{\mathcal{W}}$. Figure 4 shows the evolution of θ_0, θ_1, and θ_2 as the end effector traces the closed path $\mathcal{C}_{\mathcal{W}}$ in the workspace. It can be seen that the drift in the configuration variables is negligible.

VI. Conclusions

We discussed the repeatability problem in free-flying planar space robots. We showed that there exist certain closed trajectories in the joint space as well as in the workspace along which the space robot can exhibit holonomic behavior globally. We addressed the motion planning problem that relies on exact knowledge of the system parameters. Future research will be directed toward developing feedback control laws for repeatable motion in planar space manipulators as well as manipulators in three dimensions.

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Prescribed Pole Placement with Optimal Weight Selection for Single-Input Controllable Systems

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Introduction

THE linear quadratic regulator (LQR) has been proven to be a robust design.[1] The combination of the method of LQR and pole placement has been intensively studied in recent decades.[2–5] Reference 2 showed that standard single-input controllable systems could be optimized by solving a Lyapunov equation instead of the matrix Riccati equation. A method for optimally moving the imaginary parts of the open-loop poles to a horizontal strip was shown in Ref. 3. State feedback control law was used to assign all closed-loop poles to a specified disk centered on the negative real axis in

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